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IN THE HADRONIC CHEMICAL FREEZE-OUT

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A new way to overcome the Λ hyperon selective suppression, which is known as the Λ -anomaly, has been suggested. In particular, the additional radius of a Λ hyperon is introduced into the model of hadron resonance gas with the multicomponent hard-core repulsion. The proposed approach allows one to describe the hadron multiplicity ratios measured at the AGS, SPS and RHIC energies with the accuracy $\chi^2/dof = 52/55 \simeq 0.95$.

Keywords: hadron multiplicities, hard-core repulsion, chemical freeze-out, strangeness suppression.

1. Introduction

A number of extremely complicated experiments devoted to collisions between heavy ions and aimed at studying the properties of a strongly interacting matter under extreme conditions were carried out during last decades. For instance, the main objective of experiments carried out on the RHIC and LHC accelerators is the research of the properties of a strongly interacting matter under extreme conditions and searches for a new state of the QCD matter, a quark-gluon plasma. The theoretical efforts made in the framework of the lattice QCD and phenomenological approaches are also aimed at constructing the corresponding phase diagram. Combining together hydrodynamic, kinetic and thermal models – each of them cannot describe the collision process completely, but only one of its stage – all stages of the system evolution can be reproduced. For instance, the stage of chemical freeze-out (CF) is described by the hadron resonance gas model (HRGM) [1–3]. The authors of cited works consider the CF as a stage of collisions between heavy ions, at which all particles do not interact with one another inelastically, so that the hadron multiplicities can change only owing to decays. In addition, the HRGM is based on the assumption of a thermal equilibrium in the system. Therefore, the hadron yields are completely governed by the equilibrium parameters of CF, namely, the temperature T and the chemical potentials μ_B , μ_S and μ_{I_3} , which correspond to the conservation of

the baryon charge, strangeness and third isospin projection, respectively. The relevance of this approach to the description of particle yields was demonstrated for the collision energies in the interval from AGS to LHC [1–5].

It should be noted that, till now, the HRGM had substantial problems concerning the description of hadrons containing (anti)strange quarks. In turn, this circumstance did not make it possible to describe the available experimental data with a high accuracy. It is especially actual for the dependence of such multiplicity ratios as K^+/π^+ (the Strangeness Horn) and Λ/π^- on the collision energy.

Traditionally, in order to improve the description of all strange hadrons, the strangeness suppression factor γ_s is used; it was proposed in work [6] as a free parameter in simulations. This parameter describes a deviation of the (anti)strange hadrons from the chemical equilibrium in the system. However, it turned out that the inclusion of the parameter γ_s into the model with the one-component hard-core repulsion did not give rise to a substantial improvement of the description of hadron multiplicities [7, 8]. Just the application of the HRGM with multicomponent repulsion [4] made it possible to describe the experimental data with a high accuracy ($\chi^2/dof \simeq 1.06$) and demonstrate the importance of the strangeness suppression factor application. Moreover, unlike previous results [7], the strangeness enhancement rather than its suppression was revealed at low energies [4, 9]. Although the approach on the basis of the parameter γ_s has already been discussed for more than 20 years, its physical meaning was found only recently in works

[4, 10, 11] using the approach of separated CFs for strange and non-strange hadrons. As was shown in work [9], if only one CF is considered for all hadrons in the system, we will inevitably obtain $\gamma_s > 1$. This result really testifies to the absence of a chemical equilibrium between strange and all other hadrons, especially at low collision energies, as well as to the necessity of introducing the separate freeze-outs for them.

Nowadays, the application of γ_s fit remains the simplest and the most effective way to describe the available data. In addition, it describes the Strangeness Horn more accurately. Therefore, in this work, just the approach developed on the basis of the free parameter γ_s is applied.

As was shown in work [3], a multicomponent repulsion of the hard-core type is a necessary element of the HRGM. The introduction of the corresponding radii of hadrons takes the repulsion between constituents into account, whereas the attraction between them is taken into consideration with the help of many sorts of hadrons. An adequate way of introducing the hard-core repulsion is the consideration of hadron gas as a multicomponent mixture of particles with different radii [3, 4, 12, 13]. In order to provide the best description for all data, the baryon, R_b and meson, R_m , radii were fixed in work [4] at values of 0.2 and 0.4 fm, respectively, whereas the radii of kaons, R_K and π -mesons, R_π , were fitted independently. As a result, a high-quality fitting of experimental data was carried out with the help of the multicomponent HRGM and the free parameter γ_s , with $\chi^2/dof \simeq 1.15$ for 111 independent ratios between hadron multiplicities measured at 14 values of collision energy in the center-of-mass system ranging from 2.7 to 200 GeV. An especially considerable improvement of the description was obtained for the energy dependence of the ratio K^+/π^+ with the accuracy $\chi^2/dof = 3.3/14$ (cf. with the previous value $\chi^2/dof = 7.5/14$ obtained in work [3]).

At the same time, the theoretical description of experimental data for the Λ/π^- and $\bar{\Lambda}/\pi^-$ ratios is not satisfactory. For instance, one of the best descriptions of the ratio Λ/π^- was carried out in work [14] with the accuracy $\chi^2/dof = 10/8$. Difficulties with the description of the ratios that include the Λ and $\bar{\Lambda}$ hyperons are not new. As was marked in works [2, 3, 14, 15], a too slow decrease of the model data description for the ratio Λ/π^- is a consequence of the $\bar{\Lambda}$ -anomaly,

which was detected in works [2, 16]. Similar conclusions about the selective suppression of the yields of the \bar{p} , $\bar{\Lambda}$ and $\bar{\Xi}$ multiplicities were also drawn in works [5, 17, 18]. To solve this problem, I propose to introduce the hard core radius of the Λ hyperon and, in such a way, to account for the peculiarities of its interaction in comparison with all other hadrons. As will be demonstrated below, this supplement to the HRGM makes it possible to substantially improve not only the description of the most problematic ratios between the particle multiplicities, but also the general description of all other ratios measured in a wide energy interval from AGS to RHIC.

The structure of the work is as follows. The next section contains the basic concepts of the HRGM. Section 3 is devoted to the consideration of experimental data, which are used in this work. The obtained fitting results and some speculations concerning the improvement of the hadron multiplicity description are presented in Section 4. Section 5 with summarizing conclusions ends the paper.

2. Hadron Resonance Gas Model

Hadron multiplicities are described by means of the multicomponent HRGM, which is one of the best thermal models at present. As was shown in works [1–3, 7, 12], the quantum statistics can reasonably be neglected at corresponding temperatures of the hadron gas and the repulsion between the constituents can effectively be described with the help of hard-core radii. At the same time, the attraction between hadrons is described, as was done in the statistical bootstrap model [19], by means of a considerable number of hadronic degrees of freedom. For the effective description of hadron multiplicities, the hard-core radii of pions, kaons, all other mesons, Λ hyperons and all other hadrons were fitted.

For the thermodynamic description of the hadronic CF, a Boltzmann gas consisting of s sorts of hadrons in the volume V and at the temperature T is considered. The number of hadrons of the i -th sort will be characterized by the variable N_i , so that the total number of particles equals $M = \sum_{i=1}^s N_i$. For any two types i and j of interacting particles, we introduce the excluded volume $b_{ij} = \frac{2\pi}{3}(R_i + R_j)^3$, which enters, in turn, the matrix of virial coefficients $B = (b_{ij})$. This matrix is symmetric, i.e. $b_{ij} = b_{ji}$.

The canonical partition function for a mixture of van der Waals gases with multicomponent repulsion

looks like [12]

$$Z_{VdW}(T, V, N_i) = \left[\prod_{i=1}^s \frac{\phi_i^{N_i}}{N_i!} \right] \left[V - \frac{N^T B N}{M} \right]^M, \quad (1)$$

where N is the column matrix,

$$N = \begin{pmatrix} N_1 \\ N_2 \\ \vdots \\ N_s \end{pmatrix}, \quad (2)$$

and N^T is the corresponding transposed matrix. The one-particle thermal density $\phi_i(T, m, g)$ corresponding to the hadron of the mass m_i and the degeneration factor g_i is determined from the equation

$$\phi_i(T) = \frac{g_i}{(2\pi)^3} \int \exp \left(-\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3k. \quad (3)$$

Each i -th sort of hadrons is characterized by the total chemical potential $\mu_i \equiv Q_i^B \mu_B + Q_i^S \mu_S + Q_i^{I_3} \mu_{I_3}$, which is expressed in terms of the baryon chemical potential μ_B , the chemical potential of the third isospin projection μ_{I_3} , the strange chemical potential μ_S and the corresponding charges Q_i^L ($L = B, S, I_3$).

Since the number of particles does not remain constant at the collisions of heavy ions, it is necessary to use the grand canonical ensemble with the partition function

$$\mathcal{Z} = \sum_{N_1=1}^{\infty} \sum_{N_2=1}^{\infty} \dots \sum_{N_s=1}^{\infty} \left(\prod_{i=1}^s \exp \left[\frac{\mu_i N_i}{T} \right] \right) Z_{VdW}. \quad (4)$$

In the thermodynamic limit within the method of maximum term [20], the partition function (4) can be substituted by the term that gives the largest contribution to \mathcal{Z} . Let it be the matrix N^* . Then the pressure in the system will be determined by the relation

$$\begin{aligned} p/T &= \lim_{V \rightarrow \infty} \frac{\mathcal{Z}}{V} = \\ &= \lim_{V \rightarrow \infty} \frac{1}{V} \ln \left[\prod_{i=1}^s \frac{A_i^{N_i^*}}{N_i^*!} \left(V - \frac{(N^*)^T B N^*}{M^*} \right)^{M^*} \right], \end{aligned} \quad (5)$$

where $A_i = \phi_i \exp \left[\frac{\mu_i}{T} \right]$. In order to find N^* , let us apply the condition of the function maximum ($i = 1, \dots, s$),

$$\frac{\partial}{\partial N_i^*} \left[\ln \left[\prod_{i=1}^s \frac{A_i^{N_i^*}}{N_i^*!} \left(V - \frac{(N^*)^T B N^*}{M^*} \right)^{M^*} \right] \right] = 0. \quad (6)$$

Differentiating this formula and making the substitution $\xi_i = \frac{N_i^*}{V - \frac{(N^*)^T B N^*}{M^*}}$, we obtain

$$\xi_i = A_i \exp \left(-\sum_{j=1}^s 2\xi_j b_{ij} + \frac{\xi^T B \xi}{\sum_{j=1}^s \xi_j} \right). \quad (7)$$

The variable ξ is a column vector of corresponding coefficients ξ_i , similarly to Eq. (2).

The quantity $T\xi_i$ is nothing else but the partial pressure of hadrons of the i -th sort. Hence, using Eq. (7), the hadronic density $n_i = \frac{N_i^*}{V}$ and the system pressure p can be expressed as follows:

$$p = T \sum_{i=1}^s \xi_i, \quad (8)$$

$$n_i^L = \frac{Q_i^L \xi_i}{1 + \frac{\xi^T B \xi}{\sum_{j=1}^s \xi_j}}. \quad (9)$$

Equations (7) and (8) allow one to find the pressure in the system and Eq. (9) gives the thermal multiplicity of charges Q_L , provided that the values of T and μ_i are known.

The key fitting parameters of the model are the temperature T , the baryon chemical potential μ_B and the chemical potential of the third isospin projection μ_{I_3} , whereas the strange chemical potential μ_S is determined from the condition that the total strangeness in the system equals zero. The dependences of model parameters on the collision energy are shown in Fig. 1 for T , μ_B and μ_{I_3} . A more detailed description of the model can be found in works [3, 13].

In this work, we also consider the possible deviation of strange particles from the equilibrium. The consideration is carried out in the framework of the conventional procedure, namely, the introduction of the strangeness suppression factor γ_s . The corresponding mathematical implementation consists in the following substitution of the one-particle thermal density

$$\phi_i(T) \rightarrow \phi_i(T) \gamma_s^{s_i}, \quad (10)$$

where s_i equals the total number of strange valence quarks and antiquarks. This is the standard procedure [6] that makes it possible to consider a probable

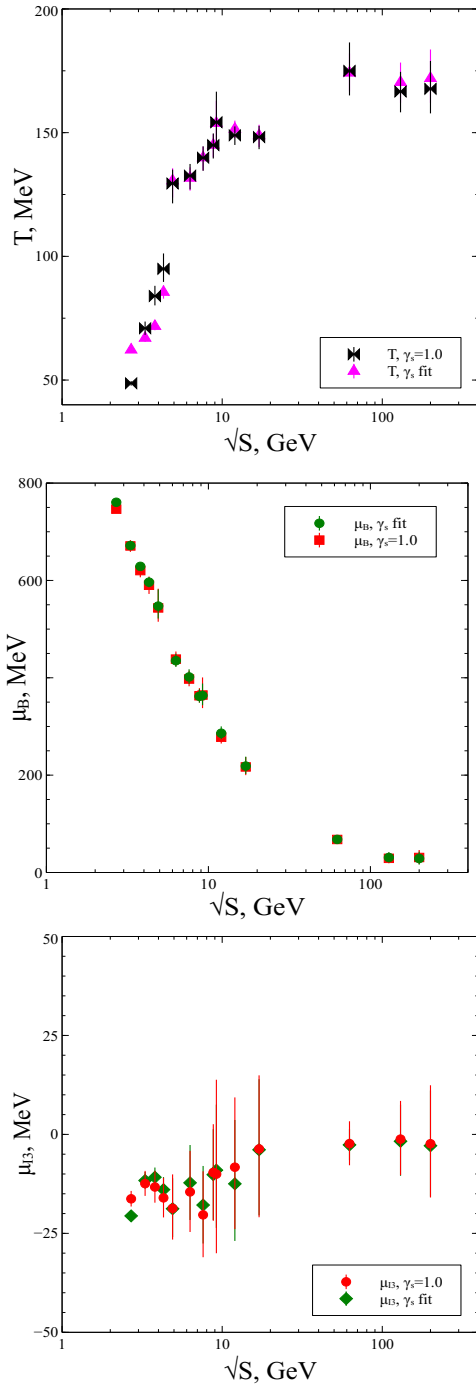


Fig. 1. Behavior of the chemical freeze-out parameters in the models with the constant ($\gamma_s = 1$) and fitted parameter γ_s : the dependences of the chemical freeze-out temperature (upper panel), the baryon chemical potential (middle panel) and the chemical potential of the third isospin projection (lower panel) on the collision energy

deviation of the strange charge from the total chemical equilibrium.

Making allowance for the width of a hadronic state, Γ_i , is one of the important elements of this model. As was demonstrated in works [3, 21], the thermodynamic properties of a hadronic system are extremely sensitive to this width. Therefore, the finite widths of resonances were introduced by means of the standard modification of the one-particle thermal density $\phi_i(T)$ [2]; namely, by averaging all the terms that include the mass over the Breit–Wigner distribution function and using the threshold M_i for each resonance. As a result, the modified one-particle thermal density for the i -th sort of hadrons takes the form

$$\int \exp\left(-\frac{\sqrt{k^2 + m_i^2}}{T}\right) d^3k \rightarrow \frac{\int_{M_i}^{\infty} \frac{dx}{(x-m_i)^2 + \Gamma_i^2/4} \int \exp\left(-\frac{\sqrt{k^2 + x^2}}{T}\right) d^3k}{\int_{M_i}^{\infty} \frac{dx}{(x-m_i)^2 + \Gamma_i^2/4}}, \quad (11)$$

where m_i is the average hadron mass.

Experimentally detected hadron multiplicities are sums of a thermal component and a component resulting from hadron decays. For instance, many pions appear owing to heavy hadron decays. The effect of resonance decays $Y \rightarrow X$ to final hadron multiplicities is taken into account as follows:

$$n^{\text{fin}}(X) = \sum_Y BR(Y \rightarrow X) n^{\text{th}}(Y), \quad (12)$$

where $BR(Y \rightarrow X)$ is the probability that hadron Y decays into hadron X . In addition, it is supposed for convenience that $BR(X \rightarrow X) = 1$. All the parameters used in the fitting of data (the masses m_i , the widths Γ_i , the degeneration factors g_i and the probabilities of decays for all strong decay channels) were taken from the particle tables of the thermodynamic code THERMUS [22].

3. Experimental Data

The model described above was used to fit the experimental data on the collisions between heavy ions, namely, the ratios of particle yields measured at the midrapidity. Unlike the particle yield fit, the fit of particle yield ratio allows us to exclude the effective volume of the system and, hence, to reduced the number of model parameters [4].

In particular, high-quality data are available for the AGS collision energies within the interval $\sqrt{s_{NN}} = 2.7 \div 4.9$ GeV (the kinetic beam energy equals from 2 to 10.7 AGeV). For energies of 2, 4, 6 and 8 AGeV, there are data on the pion [23,24], proton [25,26], and kaon (except for an energy of 2 AGeV) [24] yields, as well as on the Λ hyperon yield integrated over the angle of 4π [27]. For a beam energy of 6 AGeV, the multiplicities of Ξ^- hyperon integrated over 4π were also measured [28]. According to work [2], the yields of Λ and Ξ^- hyperons experimentally measured at the midrapidity do not meet the requirements; instead, corrected data from work [2] are considered. For the energy $\sqrt{s_{NN}} = 4.9$ GeV in the center-of-mass system, the yields of ϕ meson [29], and Λ and $\bar{\Lambda}$ hyperons [16, 30] are also available. Following work [3], I used the results for particle multiplicities at the midrapidity experimentally measured by the NA49 Collaboration [31–36]. Since the results obtained by different collaborations on the RHIC accelerator for the collisions between high-energy heavy ions coincide with one another, the STAR Collaboration data for $\sqrt{s_{NN}} = 9.2$ GeV [37], 62.4 GeV [38], 130 GeV [39–42] and 200 GeV [42–44] were analyzed.

The minimum of the relative deviation of the fit from experimental data, $\chi^2 = \sum_i \frac{(r_i^{\text{theor}} - r_i^{\text{exp}})^2}{\sigma_i^2}$, where r_i^{exp} is the experimental yield of the i -th particle, r_i^{theor} the corresponding theoretically predicted value and σ_i the total error of the experimental value, was the criterion of the best description of experimental data. In order to determine the corresponding values of hard-core radii that satisfy the criterion of root-mean-square deviation minimum, the global fitting of experimental data was made. The obtained hard-core radii of hadrons were fixed and a new fitting procedure was carried out once more. Hence, while calculating the number of the degrees of freedom in the model, the particle radii were not taken into account (this is a usual practice [1–5]), because they were previously determined and fixed.

4. Fitting Results

The best description of experimental data for 14 collision energies $\sqrt{s_{NN}} = 2.7, 3.3, 3.8, 4.3, 4.9, 6.3, 7.6, 8.8, 9.2, 12, 17, 62.4, 130$ and 200 GeV, which correspond to the χ^2 minimum, was obtained at $R_b = 0.355$ fm, $R_m = 0.4$ fm, $R_\pi = 0.1$ fm, $R_K = 0.38$ fm and $R_\Lambda = 0.11$ fm. In addition, a weak

dependence of model parameters on the pion radius was revealed, in contrast to the variation of the Λ hyperon radius.

The fitting of experimental data in the case $\gamma_s = 1$ testifies to an insignificant improvement of the description, $\chi^2/dof = 75.49/69 \simeq 1.09$, in comparison with a similar fitting for four hard-core radii, $\chi^2/dof = 80.5/69 \simeq 1.16$ [4]. Let us consider all the most significant improvements in the description of hadron multiplicities in more details. At the collision energies $\sqrt{s_{NN}} = 2.7, 3.3, 3.8$, and 4.3 GeV, the data description made in work [2] was already perfect and a further improvement did not take place, because the number of experimentally measured ratios was equal only to 4, 5, 5 and 5, respectively; and only kaons and Λ hyperons are composed of strange quarks. For the AGS energy $\sqrt{s_{NN}} = 4.9$ GeV, the introduction of the additional radius R_Λ considerably improved the description of the ratios K^-/K^+ and p/π^- , but worsened the description of the ratios K^+/π^+ and Λ/π^- (see details in Fig. 2). Substantial improvements in the description of Λ and $\bar{\Lambda}$ hyperons are observed at the energies $\sqrt{s_{NN}} = 6.3, 8.8, 12, 17, 130$ and 200 GeV. One can see from Fig. 2 that the description quality became considerably higher for the ratios including Λ and $\bar{\Lambda}$ hyperons (Λ/π^- , $\bar{\Lambda}/\pi^-$ and $\bar{\Lambda}/\Lambda$). However, the description of the ratios that include the kaon yields became worse (not regularly). For instance, the quality of the description of the K^+/π^+ , K^-/K^+ and φ/K^+ ratios became worse insignificantly at $\sqrt{s_{NN}} = 12$ GeV.

Regarding γ_s as a free fitting parameter, the yield ratios can be described better for all particles with the accuracy $\chi^2/dof = 52/55 \simeq 0.95$. As is shown in Fig. 3, the parameter γ_s exceeds 1 at low energies, which confirms the strangeness enhancement revealed in work [4]. According to Fig. 3, the dependence of the parameter γ_s on $\sqrt{s_{NN}}$ has a local minimum in the energy interval 4.3–4.9 GeV, which may testify to a qualitative variation in the system properties [21]. The search for irregularities of this type is important, because they can signal of the deconfinement phase transition. For the statement on the irregularities in this energy interval to be more exact, it is necessary to carry out a detailed experimental scan in the range of collision energies from 4 to 5 GeV in the center-of-mass system.

Considering the quality of the description of particle ratios for the fitting with γ_s , we may assert that

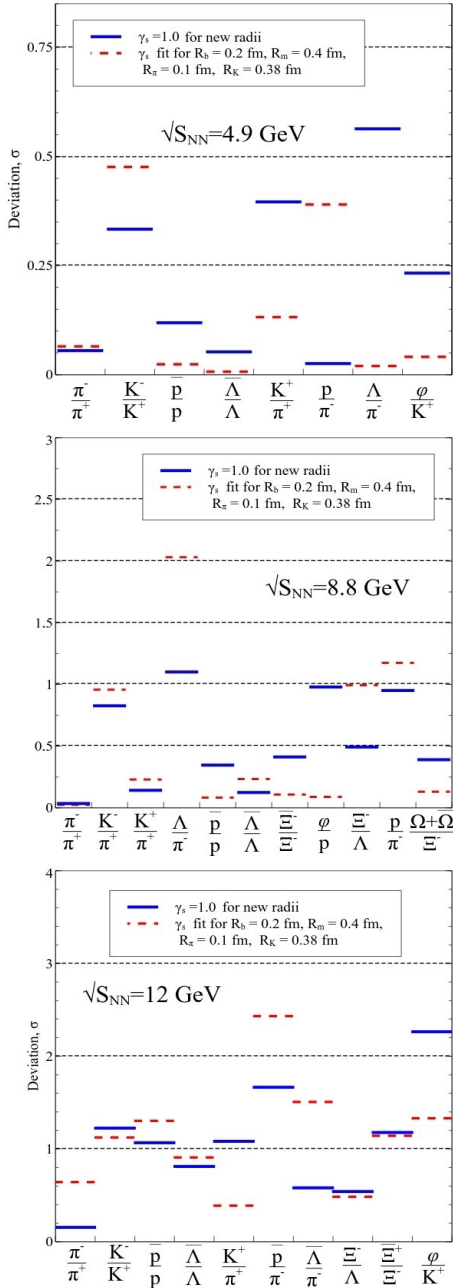


Fig. 2. Relative deviations of the theoretical description of hadron multiplicities from the corresponding experimental values reckoned in the experimental error σ units for $\sqrt{s_{NN}} = 4.9, 8.8$ and 12 GeV (panels from upper to bottom). The particle ratios are reckoned along the abscissa axis, and the absolute values of the relative deviation $\frac{|r^{\text{theor}} - r^{\text{exp}}|}{\sigma^{\text{exp}}}$ along the ordinate one. The solid lines correspond to $\gamma_s = 1$ and the dashed ones to the fit results with the free γ_s parameter obtained in work [3]

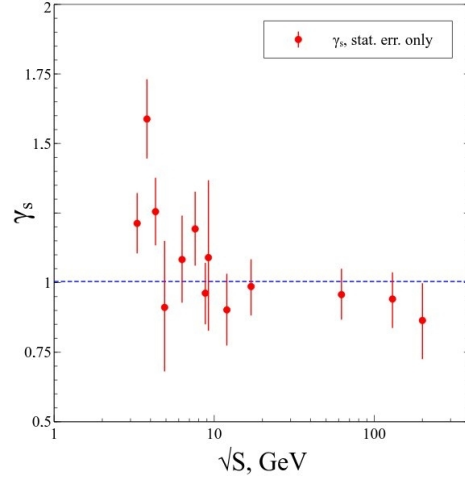


Fig. 3. Dependence $\gamma_s(\sqrt{s_{NN}})$

a general improvement of the description of particle multiplicities is observed, especially those including Λ hyperons. For the energy $\sqrt{s_{NN}} = 4.9$ GeV, unlike the previous fitting result obtained for $\gamma_s = 1$, the descriptions of the K^+/π^+ , K^-/K^+ and p/π^- ratios are improved, which is an additional argument for the necessity of using the γ_s factor. Figure 4 gives some examples of the improved theoretical description of hadron multiplicities at $\sqrt{s_{NN}} = 4.9, 8.8$ and 12 GeV.

An important result is the improvement of the description of the $\sqrt{s_{NN}}$ dependences for the ratios that are the most problematic in the HRGM. For instance, the description accuracy for the dependence of the Λ/π^- ratio on the collision energy becomes higher for two fitting procedures – namely, for $\gamma_s = 1$ ($\chi^2/dof = 14.48/12$) and for γ_s taken as a free parameter ($\chi^2/dof = 10.22/12$) – in comparison with the highest accuracy of the previous description attained in work [4] ($\chi^2/dof = 14.85/12$). Figure 5 illustrates the result of high-quality fit carried out for the dependence of the Λ/π^- ratio on the collision energy. The corresponding accuracy $\chi^2/dof = 6.49/8$ was obtained for the free parameter γ_s , which is better than the accuracy $\chi^2/dof = 10/8$ obtained in work [14]. The fit of the Strangeness Horn shows an insignificant worsening in comparison with its best description ($\chi^2/dof = 1.5/14$) in work [14]; however, the description quality remains high enough, because it can completely describe the dependence maximum. The description accuracy for the $\sqrt{s_{NN}}$

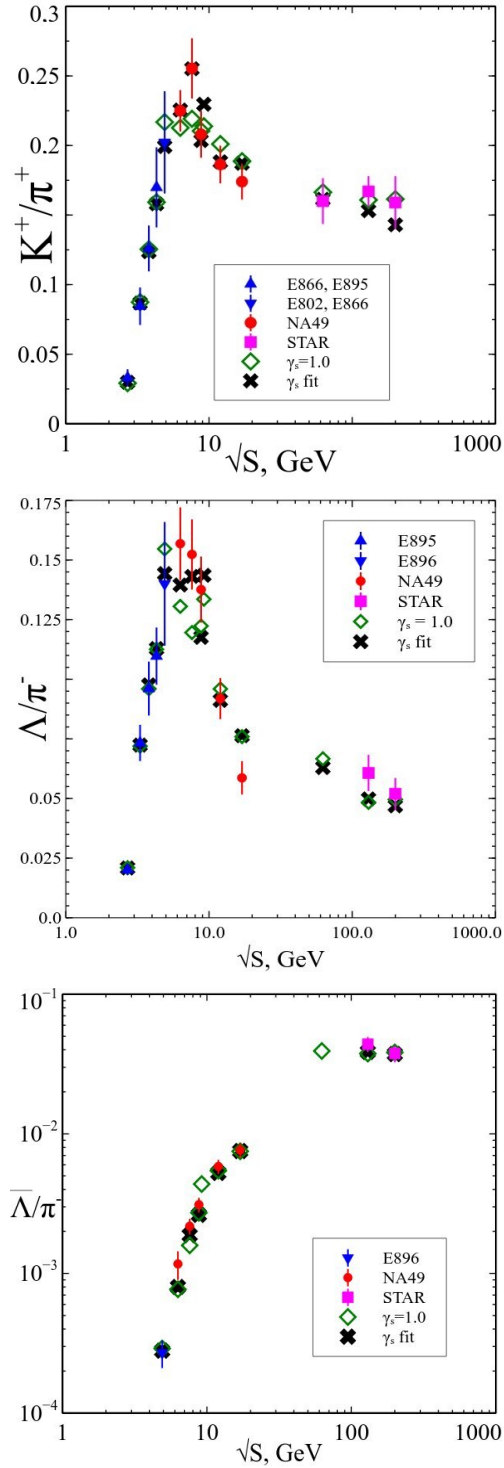


Fig. 4. Dependences of the K^+/π^+ (upper panel), Λ/π^- (middle panel) and $\bar{\Lambda}/\pi^-$ (bottom panel) ratios on $\sqrt{s_{NN}}$ for the constant ($\gamma_s = 1$) and fitted parameter γ_s

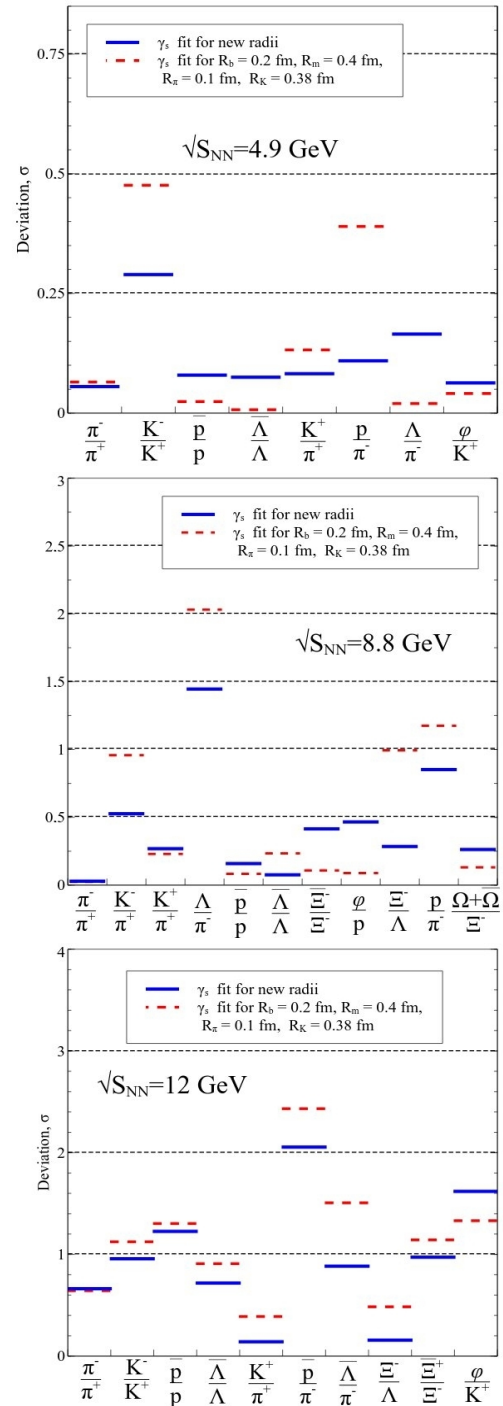


Fig. 5. Relative deviations of theoretical hadron multiplicity descriptions obtained for the free γ_s fit parameter from their experimental values reckoned in the experimental error σ units and their comparison with the results of fitting in work [3] also obtained for the free γ_s parameter

dependence of the K^+/π^+ ratio amounts to $\chi^2/dof = 7.25/14$ in the case of the fixed γ_s factor and to $\chi^2/dof = 3.92/14$ in the case of the free γ_s parameter.

Despite that the ratio χ^2/dof only slightly differs between the cases with the fixed or free parameter γ_s – namely, 1.09 and 0.95, respectively – the χ^2 -value itself decreased from 75 to 52, which testifies to a substantial improvement of the data description. Hence, a conclusion follows that, in the cases when the difference between the values of χ^2/dof is insignificant, it is necessary to introduce an additional criterion of the data description quality. As such a criterion, the description quality of Λ/π^- and $\bar{\Lambda}/\pi^-$ multiplicity ratios was selected. The appreciable improvement of the description for the most problematic ratios in the HRGM is an important argument in favor of γ_s for the QCD phenomenology. The γ_s -values found at low energies testify to the absence of a chemical equilibrium between the strange and all other hadrons, which is explained by the hypothesis of separate chemical freeze-outs [4, 9, 11].

5. Conclusions

The influence of the hadron hard-core radius on the description quality of experimental hadron yields has been systematically analyzed in the framework of the multicomponent hadron resonance gas model. The hard-core radii found for baryons, $R_b = 0.355$ fm, mesons, $R_m = 0.4$ fm, pions, $R_\pi = 0.1$ fm, kaons, $R_K = 0.38$ fm and Λ hyperons, $R_\Lambda = 0.11$ fm, satisfy the condition of χ^2 minimum and provide the best quality of the description of experimental data. Those improvements of the model allowed us to obtain a high-quality fit of experimental data measured in the collision energy interval from AGS to RHIC ($\sqrt{s_{NN}} = 2.7 \div 200$ GeV). The introduction of the additional hard-core radius for the Λ hyperon essentially improved the description of the dependences of Λ/π^- and $\bar{\Lambda}/\pi^-$ ratios on the collision energy with the accuracy $\chi^2/dof = 10.22/12$ and $6.49/8$, respectively. The description of the Strangeness Horn shows the absolute correspondence between experimentally and theoretically determined points with $\chi^2/dof = 3.92/14$. At the collision energies $\sqrt{s_{NN}} = 3.3, 3.8, 4.9, 6.3, 7.6$, and 8.8 GeV, the value $\gamma_s > 1$ was obtained, which corresponds to the strangeness enhancement. As is seen from Fig. 3, a local minimum of the parameter γ_s was revealed in the inter-

vals of collision energies in the center-of-mass system from 4.3 to 4.9 GeV, which can be an additional argument in favor of qualitative changes in the system properties at those energies [21]. The applied approach made it possible to describe the ratios between all hadron multiplicities with the accuracy $\chi^2/dof = 52/55 \simeq 0.95$, which is the best one at present.

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